33230 Mathematical Modelling 2: Assignment 1 (Mathematics)

This assignment is to be done in groups of 1, 2 or 3 students. You need to sign and attach the assignment submission sheet and hand in only one assignment per group. Drop off assignment in the box next to the elevators, level 15 of Tower building by ***5:00pm, Monday, 1st September 2008***. The assignment will be marked out of ***5***. Note: there is a bonus mark for this assignment it is possible to obtain 6 marks.

The purpose of this assignment is to further your skills and understanding of matrices and linear transformations. You must show all working where relevant. If using *Mathematica* for any part, include all input code and output as your working. You must acknowledge and reference any additional sources used, plagiarism will not be tolerated.

Please follow the instructions for the use of $val1$, $val2$ and $val3$.

Place your group members by surname in alphabetical order. Let $val1$ = the last nonzero digit of the first member's student number, $val2$ = the last nonzero and distinct digit of the second member's student number, and similarly for $val3$, such that all 3 values are different. If there is a repeat digit, please discard and use the previous nonzero digit of your student number.

(If there are only 2 members in your group, then switch back to the first student for $val3$. If you are doing the assignment on your own, use the second last distinct non zero digit for $val2$ and similarly for $val3$)

You should now have 3 distinct nonzero values between 1 and 9. Write down the three values.

Question 1 (1.5 marks)

(a) What is a symmetric matrix?

(b) Let $A = \begin{bmatrix} val1 & val2 \\ val3 & 1 \end{bmatrix}$. Find AA^T.

(c) For any order matrix *A*, both AA^T and $A^T A$ are symmetric. Show that this is true for any 2×2 matrix.

Question 2 (3.5 marks)

Plot the points $A(val1, val2)$, $B(val1, val3)$ and $C(val2, val3)$ on a piece of grid paper and join the points to form a right-angled triangle ΔABC. We wish to carry out the following linear transformations on the triangle.

- A $90°$ clockwise rotation, about the origin
- A reflection in the line $y = x$.

(a) Calculate the 2×2 matrix for carrying out the $90°$ clockwise rotation and apply it to the points A, B and C to obtain new points A', B' and C'. Plot these points on the same piece of grid paper to form the triangle $\Delta A'B'C'$

(b) Calculate the 2×2 matrix for carrying out the reflection in the line $y = x$ and apply it to the points A', B' and C' to obtain new points A'', B'' and C''. Plot these points on the same piece of grid paper to form the triangle $\Delta A''B''C''$

(c) Describe geometrically the linear transformation which has occurred from ΔABC to $\Delta A''B''C''$ and calculate the 2×2 matrix for carrying out this transformation.

(d) Show that a $90°$ clockwise rotation about the origin followed by a reflection in the line $y = x$ on any 2-dimensional point or vector, results in the linear transformation described in part (c).

Challenge Question (bonus 1 mark)

Complete exercise 9.7 Question 15 from text-book *Fitzgerlad and Peckham*.